Tangent Plane Calculation

Example: \( f(x, y) = x^2 + y^2 \)

Tangent plane at \((1, 2)\)?

\[
f_x(x, y) = 2x \quad f_x(1, 2) = 2
\]

\[
f_y(x, y) = 2y \quad f_y(1, 2) = 4
\]

\[
\frac{f_x(1, 2)}{2} \quad \frac{f_y(1, 2)}{4} \quad \text{always } -1
\]

Normal vector: \((2, 4, -1)\)

Point: \((1, 2, 1^2 + 2^2) = (1, 2, 5)\)

Thus, the plane is given by:

\[
(2, 4, -1) \cdot (x - 1, y - 2, z - 5) = 0
\]

\[
2(x - 1) + 4(y - 2) - (z - 5) = 0
\]

\[
2x + 4y - z = 5
\]

If we solve for \(z\):

\[
z = 2x + 4y - 5
\]

This function \(h(x, y)\) is the linearization of \(f\) at \((1, 2)\).